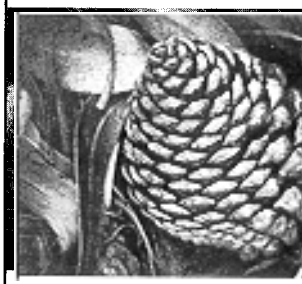
PHYLLOTAXIS



NOTE TO THE STUDENT

Every time you pick up a pine cone, look at a sunflower, eat asparagus, or cut up a fresh pineapple, you are seeing an example of the natural tendency of plants to grow in patterns that involve Fibonacci numbers. This unit on numbers and nature is written to provide you with an appreciation of how mathematics is related to the world we live in. It is hoped that an understanding of the relationship may enrich your life and make you more aware of the natural beauty around you.

The objectives of this unit are to show you how to count and record your observations of things in nature.

PRETEST

1 Give the next two numbers in the sequence.

1, 1, 2, 3, 5, 8, _____, _____

2 Which of the following exhibits Fibonacci patterns?

a cucumber **b** pineapple

c tomato **d** olive

3 What does the word *phyllotaxis* mean?

Answers

1 13, 21 **2** **b** **3** Leaf arrangement

In the year 1202, an Italian named Leonardo Fibonacci wrote a book on mathematics, titled *Liber Abaci*. One of the interesting word problems found in the book was about rabbits. He asked for the number of rabbits that would be produced in a year, starting with one pair of rabbits. For convenience, it was assumed that each pair of rabbits always produced just one pair of rabbits each month.

The answer to the problem involved the following sequence of numbers, which are called the Fibonacci numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, · · ·

The pattern for finding the next number of the sequence is very simple. We add the two preceding numbers to get the next number.

$$1 + 1 = 2$$
$$1 + 2 = 3$$
$$2 + 3 = 5$$
$$3 + 5 = 8$$
$$5 + 8 = 13$$
etc.

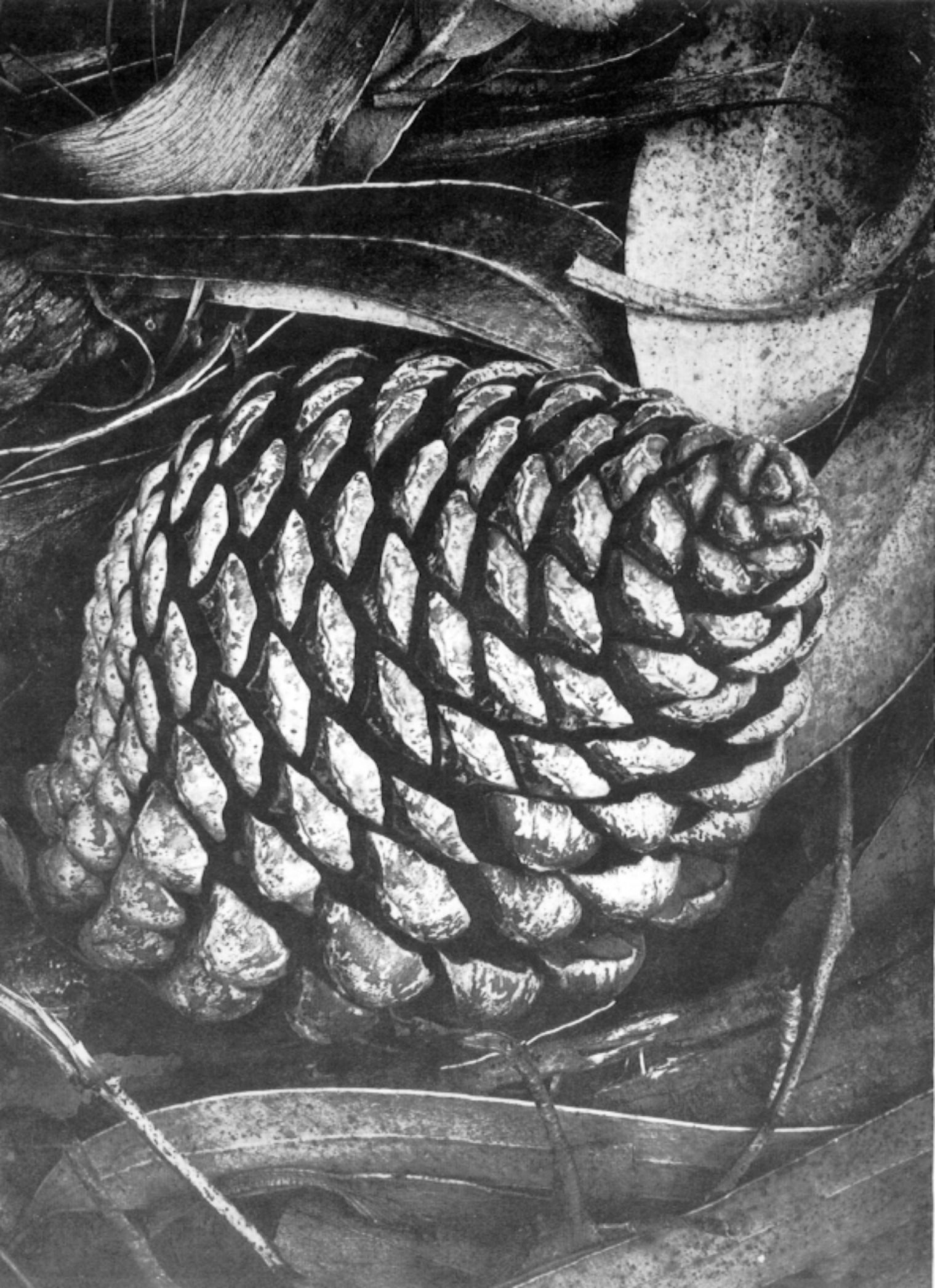
Fibonacci numbers are interesting to work with because we find the numbers turning up in some rather unexpected places. For example, we find Fibonacci numbers turning up in sunflowers, pineapples, pine cones, asparagus, and even in cactus plants.

Perhaps you have picked up a pine cone in the forest and admired its symmetry or have looked at a fresh pineapple in the supermarket and noticed the pattern of spirals. Both the pine cone and the pineapple have something in common. The natural growth of each tends to follow the Fibonacci pattern. The pine cones in particular provide a good example.

The photograph "Pine Cone and Eucalyptus Leaves" by the renowned photographer Ansel Adams shows a beautiful example of the symmetry found in a pine cone.

You should try to find a few pine cones that you can use for the unit. If you cannot get a pine cone, the Method 1 diagram will help you see the patterns; but, in order to really understand, you should find the patterns on a pine cone yourself.

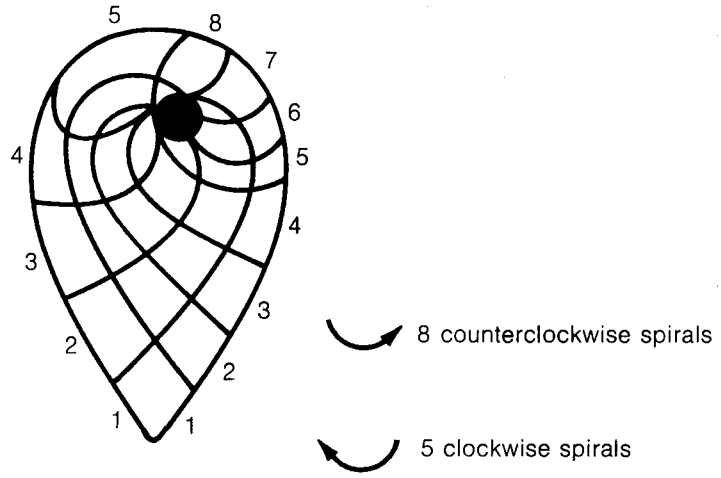
You may find it easier to work with one of the larger pine cones like the sugar pine cone. The smaller cones, like those of the lodgepole pine, are a little more difficult to work with, but can be used. Since the scales of most pine cones open if they have been on the tree a long time, and even young cones that have fallen to the ground eventually open, you will find it advisable to soak the cones in water to get the scales to close. The cone scales open naturally in late summer or autumn to release their seeds, but they will close easily when soaked in water. The patterns will be much easier to find with the scales closed. Two methods for finding Fibonacci numbers are explained on the following pages.



METHOD 1

Select one of your pine cones and look at the end of the cone that was attached to the branch of the tree. You will find that the seed-bearing scales grow outward in two sets of spirals—one clockwise and the other counterclockwise, as shown in the diagram below.

Counting Spirals on Cone



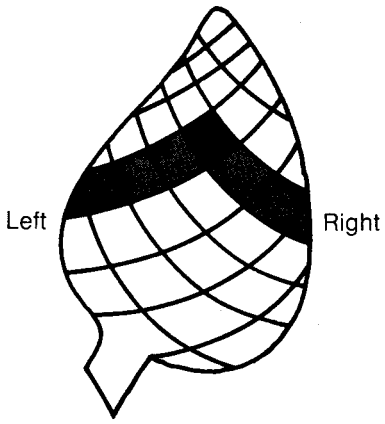
If you count the clockwise, or left, spirals in the above example, you should get five spirals and when you count the counterclockwise, or right, spirals, you should get eight spirals. Notice that 5 and 8 are consecutive Fibonacci numbers.

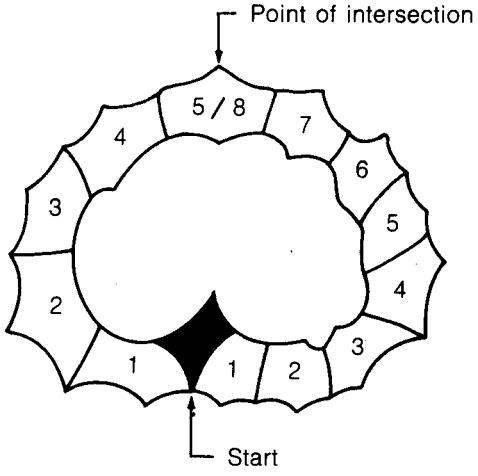
PROBLEM 1 Count the number of right and left spirals on your pine cone and record the answers in the space provided:

Number of right spirals _____ Number of left spirals _____

METHOD 2

There is a second way to find the Fibonacci pattern on a pine cone. Choose a particular scale; it does not really matter which one at this point. You should see that the scale is contained on both a right and left spiral as shown in the following diagram:





The two spirals containing the scale you chose intersect on the other side of the cone. Start at the scale you chose and count the number of scales on the right spiral until you reach the point of intersection. Repeat the process for the left spiral. Note that you do not count the scale you start with, but you do count the one where the spirals intersect. If you chose a scale too near one end of the cone and cannot find the place where the two spirals intersect, then choose a different scale and try again. Record your answer in the space provided and compare your result with the numbers you found using the first method in Problem 1.

PROBLEM 2 Number of scales on right spiral _____

Number of scales on left spiral _____

Try choosing a second scale near the end of the cone opposite the stem. Repeat the procedure of counting the scales on the right and left spirals from the scale you choose to the point of intersection. Record your result and again compare your numbers with the previous two results.

Number of scales on right spiral _____

Number of scales on left spiral _____

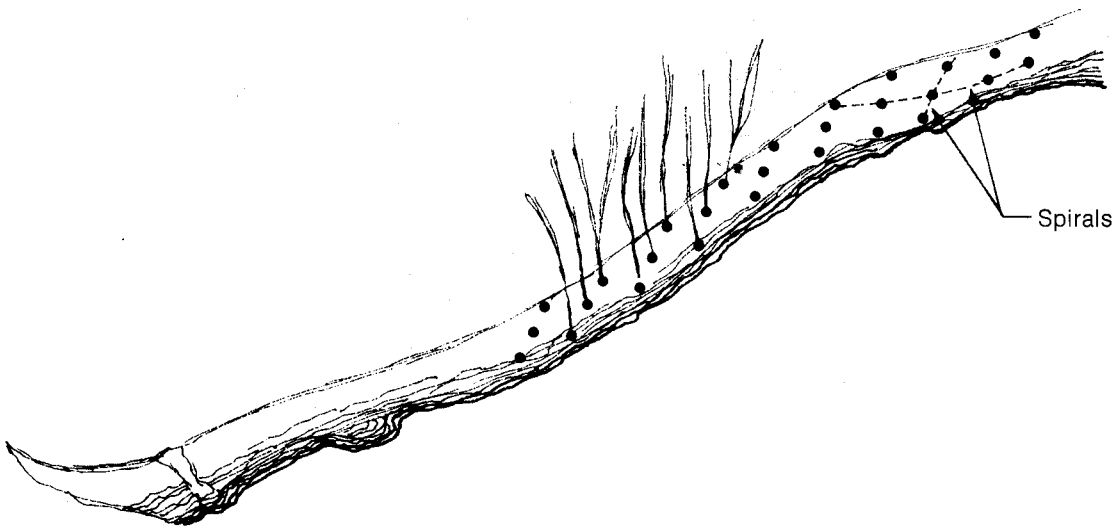
PROBLEM 3 Number of scales on right spiral _____

Number of scales on left spiral _____

You may well find that each of your three counts gave you different numbers, but each time you obtained two numbers that belonged to the Fibonacci numbers. At the tip of the cone you can often find 2 and 3 or 3 and 5 while near the stem you can find the numbers 5 and 8 or 8 and 13. Occasionally you can find the numbers 13 and 21 on a cone. The change in the growth of the cone as it matures accounts for the different numbers.

You may find that it is easier to use Method 1 on some cones because of their size. Method 1 works especially well on the Monterey pine. Also, counting the number of spirals will allow you to compare the patterns on different cones more easily as there is only one count you can get using Method 1; there are several different counts you might get using Method 2.

The leaf arrangement on pine trees also exhibits Fibonacci patterns. Obtain a branch from a pine tree and observe the spiral arrangement of the leaf bundles. This can be easily done by removing the bundles from one of the spirals. There will again be a right spiral and a left spiral, which can be treated in the same way as the scales on the pine cone. You can see the scars on the branch where the leaf bundles were removed. These scars may be counted from one of them to the point of intersection of the two spirals containing the bundle you started with on the branch. The diagram below should illustrate the idea for you. You very likely will find a different count for the leaf spirals than for the cone from the same tree, but each set of numbers will belong to the set of Fibonacci numbers.



PROBLEM 4 Write the two numbers you found on the branch from the pine tree.

ans _____ _____

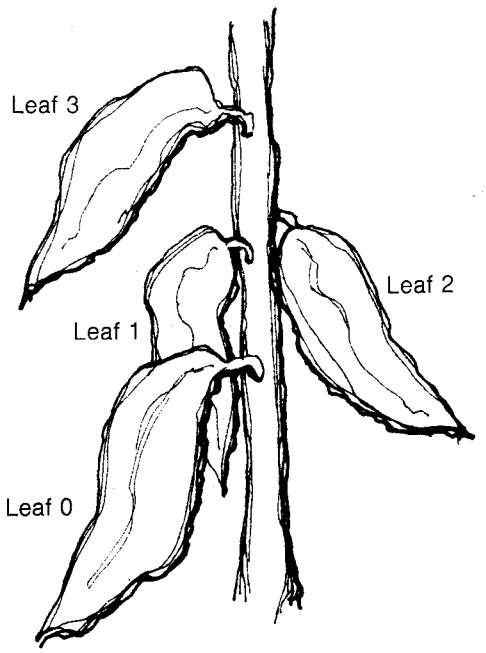
The Greek words for *leaf arrangement* are *phyllon taxis* which have been put together to form the word *phyllotaxis,* the title of this unit. There is a second way of discovering Fibonacci numbers in leaf arrangements. This method can be used on trees other than the pines and fir trees with better results than the first method.

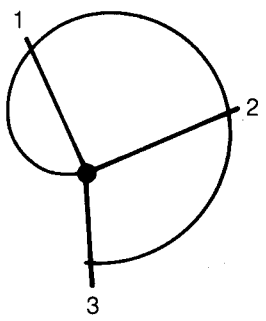
The second method involves starting with one leaf and then counting the leaves up the stem in either a clockwise or counterclockwise direction until you reach a leaf that is directly above your starting point. The number of leaves from your starting position to the one directly above (again, do not count the one you started on) usually is a Fibonacci number. Remember that there may be exceptions, so compare several branches. Now count the number of full turns, clockwise or counterclockwise, that you made from the starting point to get to the leaf directly above it. This number also usually beongs to the Fibonacci numbers. An illustration is provided of the arrangement on a peach tree. The results of the second method are usually given as a fraction:

$$\frac{\text{Number of turns}}{\text{Number of leaves}}$$

For example, the ratio found on the peach branch illustrated below would be called 1:3 or $\frac{1}{3}$ phyllotaxis.

Peach Branch Phyllotaxis





1 full turn clockwise

PROBLEM 5 Obtain branches with leaves on them from at least three different kinds of trees. Determine the phyllotaxis for each branch. Then fill in the table below, and illustrate the phyllotaxis for each branch with a drawing in the space provided.

	Name of tree	Phyllotaxis
1		
2		
3		

You have now worked with both pine cones and leaves and have seen how the growth of each results in patterns that involve Fibonacci numbers. Other plants like sunflowers, pineapples, asparagus, and cactus exhibit similar patterns.

You may find, however, that the patterns turn out to be numbers belonging to the set of Lucas numbers: 1, 3, 4, 7, 11, 18, 29, 47, 76, · · · .

Both sunflowers and pineapples may exhibit Lucas patterns as well as Fibonacci patterns.

PROBLEM 6 Find either the Fibonacci or the Lucas pattern on a pineapple. Write down the numbers you find.

ans _____

It is hoped that as a result of this unit that you will become more aware of the patterns and symmetry that are around you each day. However, remember that there are always exceptions to every rule. The patterns you have found are not universal laws. They should be accepted only as a prevalent tendency of nature.

UNIT REVIEW

1 (See 1) List five Fibonacci numbers.

2 (See 5) What does the word *phyllotaxis* mean?

3 (See 1) Describe the method of counting spirals that will give only one count on a pine cone?

4 (See 5) Which of the following would be an example of a Fibonacci pattern?

a 1,6 **b** 2,7 **c** 3,5 **d** 1,4

5 (See 6) What other pattern might you find in looking at pineapples, sunflowers, and cactus?

SUPPLEMENTARY PROBLEMS

The following two problems will expand some of the ideas of this unit.

1 Obtain as many examples of plants displaying patterns involving Fibonacci numbers as possible. Make a list indicating the most prevalent counts for each plant.

2 Classify the ratio of leaf arrangements on as many local trees in your area as you can. Again make a table of your results.

ANSWERS

Unit Review

1 1, 2, 3, 5, 8, 13, 21, etc., are Fibonacci numbers.

2 *Phyllotaxis* means leaf arrangement.

3 Count the clockwise and counterclockwise spirals at the stem end.

4 **c**

5 The Lucas pattern