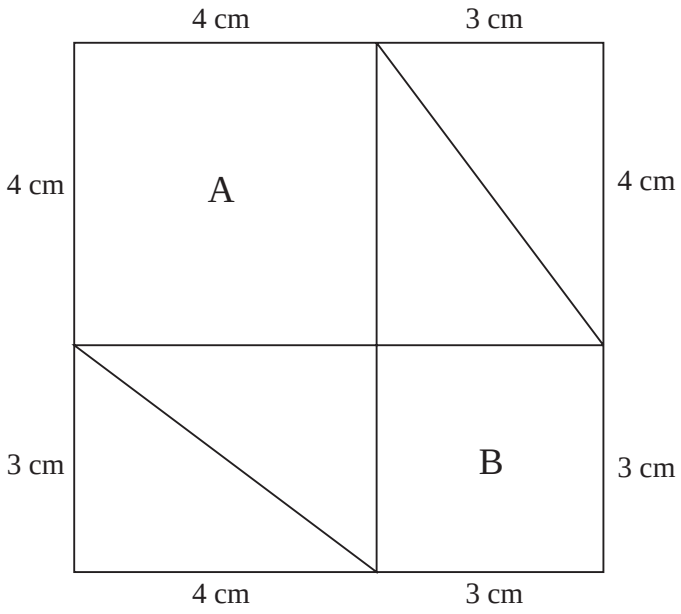
UNIT 3 *Pythagoras' Theorem* Activities

Activities

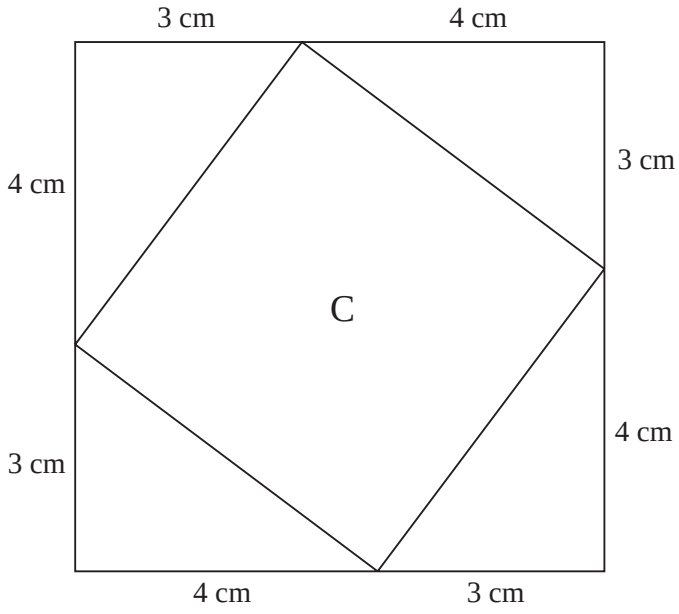
ACTIVITY 3.1.1 *Bronowski's Proof of Pythagoras' Theorem*

1. Draw the following square accurately and divide it into sections as shown:



2. Cut out the 6 parts of the square.

3. Rearrange the 4 triangles to form this square, and check that C, which should be empty, is also a square.



4. Answer the following questions:

   (a) What is the total area of the two squares, A and B, that have not been used?

   (b) What is the area of the square C inside the second square?

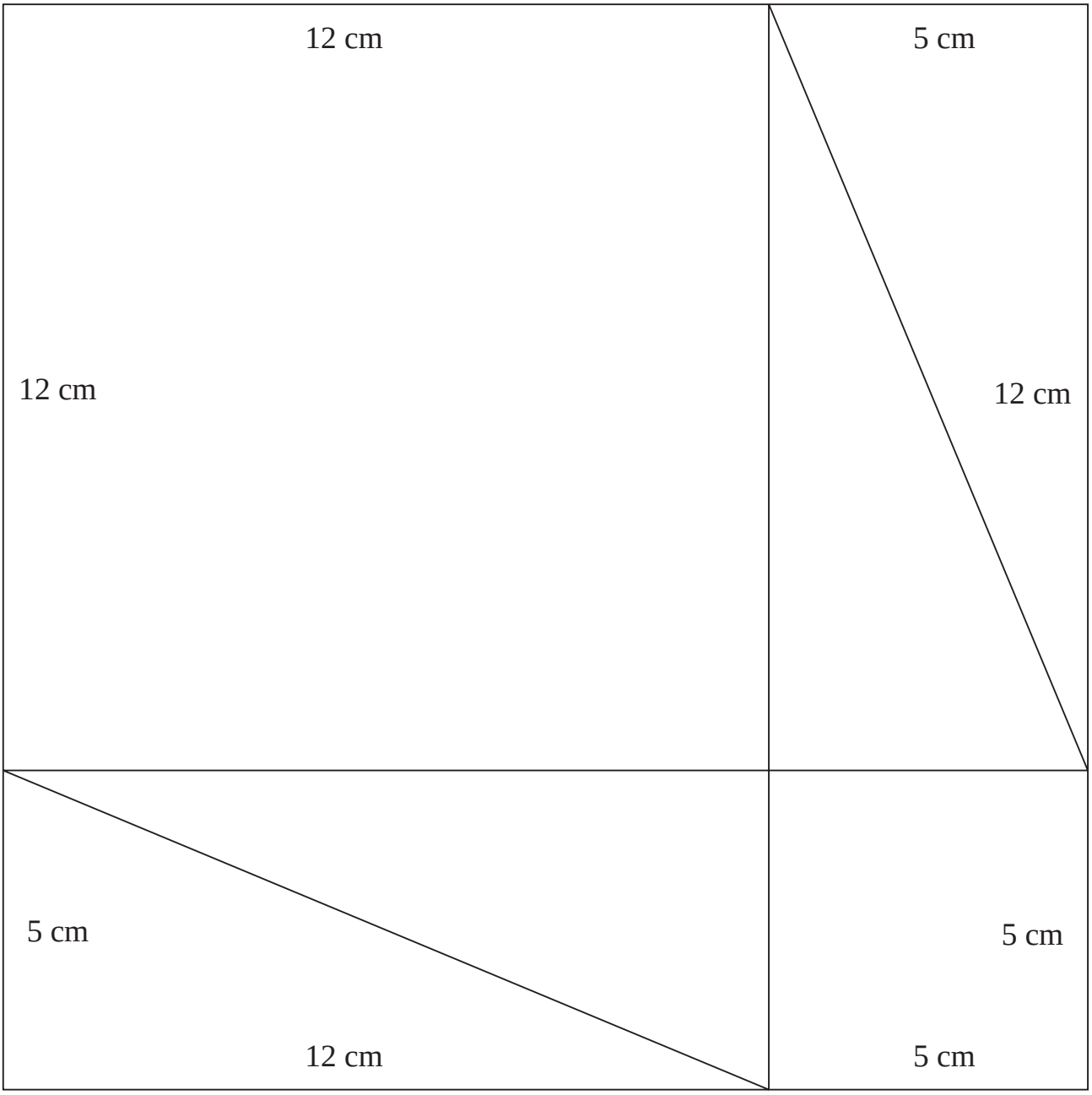
   (c) Explain why:

   Area of A + Area of B = Area of C.

   (d) Explain why the length of the hypotenuse of the triangles is 5 cm.

ACTIVITY 3.1.2 *Bronowski's Proof, continued*

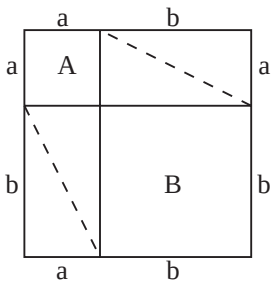
5. Start with the following shape and find the length of the hypotenuse of the triangles, by forming a square with sides the same length as the hypotenuse of the triangles.

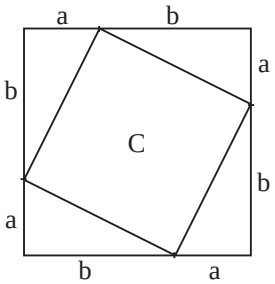


Extension

By starting with two squares as shown opposite, prove Pythagoras' Theorem,

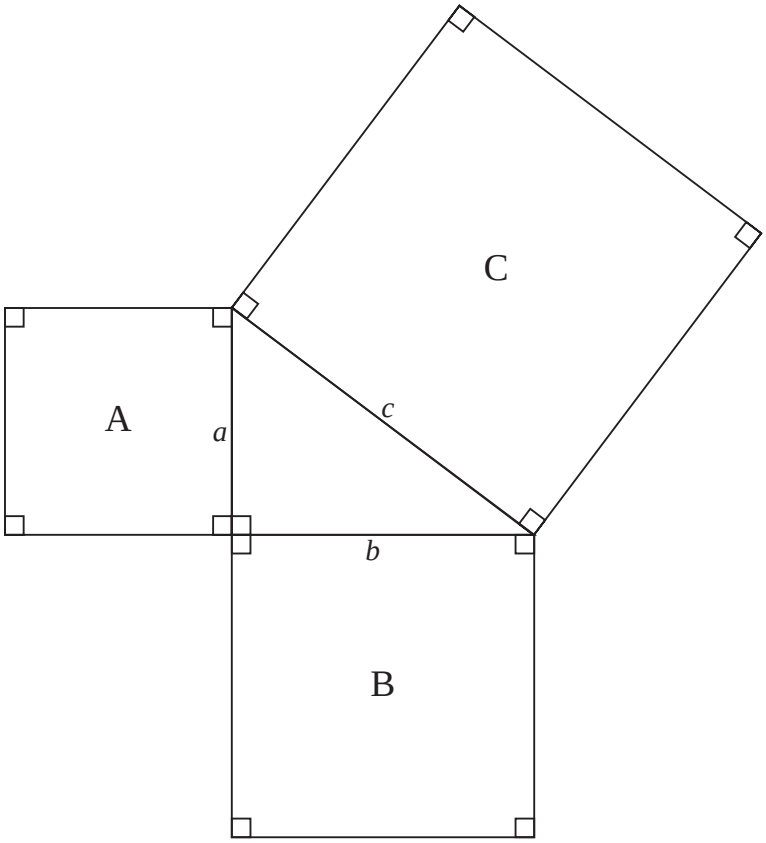
$a^2 + b^2 = c^2$.





ACTIVITY 3.2 *Squares on the Sides of a Triangle*

The diagram shows how to draw squares on each of the sides of a right-angled triangle.



1. (a) Draw a diagram like this, with $a = 3$ cm, $b = 4$ cm and $c = 5$ cm.
   (b) Calculate the areas of the 3 squares.

2. Repeat question 1 with $a = 5$ cm, $b = 12$ cm and $c = 13$ cm.

3. How does the area of the largest square relate to the areas of the smaller squares?

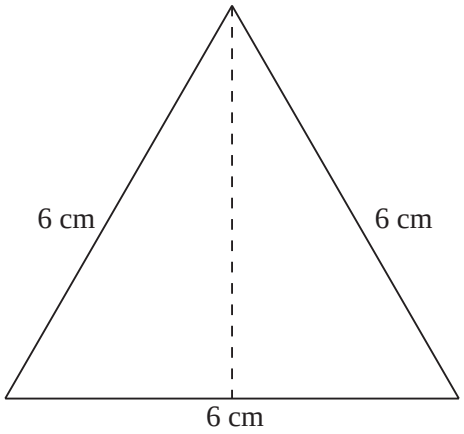
4. Copy and complete the following table:

a	*Area of* A	*b*	*Area of* B	*Area of* C	*c*
6		8			
5		4			
9		10			
	1		4		
	9			36	

ACTIVITY 3.3 *Equilateral Triangles*

1. The diagram shows an equilateral triangle with sides of length 6 cm.

   Calculate the perpendicular height of the triangle.



2. Complete the following table, for equilateral triangles:

Length of Side	*Perpendicular Height*
2	
7	
10	
15	
16	
	4
	10

3. The following rule can be used to calculate the perpendicular height of an equilateral triangle:

$$\text{perpendicular height} = k \times \text{side length}$$

   where k is a fixed number.

   Find k, correct to 2 decimal places, using the examples from the table in question 1.

ACTIVITY 3.4 *Pythagorean Triples*

Three numbers can be called a *Pythagorean triple* if

$$a^2 + b^2 = c^2$$

and all the numbers are integers (positive whole numbers).

1. Show that the following triples are all Pythagorean:

   (a) 3, 4, 5

   (b) 6, 8, 10

   (c) 5, 12, 13

   (d) 7, 24, 25

   (e) 10, 24, 26

   (f) 9, 12, 15

2. Explain why (a), (c) and (d) above are more important than the others.

3. Look at the following triples:

   3, 4, 5

   5, 12, 13

   7, 24, 25

   Note that the smallest number in the triple is an odd number.

   Note that the difference between the other numbers is 1.

   Can you find triples in which the smallest number is 9, 11 and 13 ?

ACTIVITIES 3.1 and 3.2 *Notes for Solutions*

Notes and solutions given only where appropriate.

3.1 4. (a) $3^2 + 4^2 = 9 + 16$

$= 25\ \text{cm}^2$

(b) $25\ \text{cm}^2$

(c) Outer squares (sides 7 cm) have the same area, the triangles have the same area, so area of 2 small squares is equal to area of bigger square.

area of A + area of B = area of C

(d) Area of C = $25\ \text{cm}^2$

Length of sides of C = 5 cm

Length of hypotenuse = 5 cm

5. Length of hypotenuse = 13 cm

Extension

As the four triangles are equal in area, and the two starting squares are also equal in area, then

area of A + area of B = area of C

i.e. $a^2 + b^2 = c^2$

3.2 1. (b) $9\ \text{cm}^2$, $16\ \text{cm}^2$, $25\ \text{cm}^2$

2. (b) $25\ \text{cm}^2$, $144\ \text{cm}^2$, $169\ \text{cm}^2$

3. area A + area B = area C

4.

a	*Area of* A	b	*Area of* B	*Area of* C	c
6	36	8	64	100	10
5	25	4	16	41	6.4
9	81	10	100	181	13.5
1	1	2	4	5	2.2
3	9	5.2	27	36	6

(Answers where appropriate, to 1 d.p.)

ACTIVITY 3.3 and 3.4 *Notes for Solutions*

3.3 1. 5.2 cm

2.

Length of Side	*Perpendicular Height*
2	1.7
7	6.1
10	8.7
15	13.0
16	13.9
4.6	4.0
11.5	10.0

Answers all correct to 1 decimal place.

3. $k = 0.87$, to 2 decimal places.

3.4 2. (b) and (f) are multiples of (a).

(e) is a multiple of (c).

So (a), (c) and (d) are important because they are not multiples of other triples.

3. 9, 40, 41

11, 60, 61

13, 84, 85